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The dispersion of a pressure pulse in the atmosphere

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The object is to find what pressure oscillations would be observed on the ground at a great distance from an explosion. The explosion is represented mathematically by a Fourier integral, corresponding to the introduction of a large volume into the atmosphere at a point on the ground. The resulting pulse is calculated for various distances for a model atmosphere consisting of a troposphere with a constant lapse-rate of temperature and an isothermal stratosphere. It is composed of those oscillations that can be propagated horizontally as gravity waves in this model atmosphere, namely, those of period exceeding a cut-off period of 111 sec. The pulse consists of a series of waves of decreasing amplitude and period, terminating with a period of 12·7 sec.

The results are compared with the oscillations observed on the occasion of the fall of the Great Siberian Meteorite and the energy which it is estimated to have communicated to the atmosphere is about 4×10^{24} ergs only a fraction of which resided in the gravity wave. Neglect of the warmer layers in the higher levels in the stratosphere means that the calculated pulse terminated too soon, and a second series of waves of considerable amplitude and of greater frequency is completely absent. The form of these has not been calculated because of the prohibitive amount of computing involved.

1. Introduction

The problem we seek to solve is to find what pressure oscillations would be observed at a great distance from a large explosion at the earth's surface. The amplitude and form of the oscillations, when related to the magnitude and distance of the explosion, would then provide, on comparison with observations, an estimate of the energy communicated to the atmosphere by the Great Siberian Meteorite of 1908 or the eruption of Krakatoa in 1883.

In order to render the problem reasonably simple many assumptions are made. The earth is assumed to be spherical and the atmosphere to be almost horizontally stratified—the temperature and lapse-rate of temperature and the height of the tropopause are therefore assumed to be uniform over the earth's surface and the stratosphere to be isothermal. There is assumed to be no motion other than that produced by the explosion and this is supposed to take place adiabatically and to be of small magnitude. The actual atmosphere is thus not very faithfully simulated, but apart from the wind structure, which is too complex to represent anyway, it is apparent from the equations that no great difference would result from these assumptions except from the neglect of the warmer layers of the stratosphere above about 30 km. These are known, from the work of Whipple (1935) and others, to reflect audible air waves downwards, and this fact is discussed when theory and observation are compared. These audible air waves were treated in terms of geometrical optics, the velocity of propagation being dependent only on the temperature at any point; the oscillations to be discussed here, however, are primarily gravity waves of much longer period than sound waves.

The explosion causes a localized upward displacement of the surfaces of constant density which were originally horizontal, and sends out circular wave fronts in the same way as the intrusion of an object into the surface of static water gives rise to a local elevation of the surface followed by a set of circular waves travelling outwards. The atmosphere is a dispersive medium: the phase velocity depends upon the wavelength; and so it is necessary to discover first the relationship between them. That such a relationship exists depends upon the fact that the horizontal and vertical variations of pressure (or whatever quantity is chosen to represent the disturbance) may be separated mathematically, and vertical wave fronts may be propagated horizontally over the earth's surface, the amplitude and phase of the wave at any height being determined by a differential equation relating the pressure to the height only. To this equation the boundary conditions at the 'top' and bottom of the atmosphere are applied, and when this is done the required relationship between phase velocity and frequency is obtained.

It then only remains to synthesize the explosion by means of a Fourier integral. This is in effect a boundary condition applied to the two differential equations for the vertical and horizontal variations of pressure, and it asserts that over a small region the ground rises rapidly for a short time and introduces a calculated volume into the atmosphere. The surfaces of constant density are suddenly distorted upwards and a 'pulse' moves away radially. The waves of various frequencies composing the pulse are dispersed and the pulse form changes as it travels out; the pulse is observed by the pressure variations it produces at the ground, and it is these that have been calculated for various distances from the explosion.

Taylor (1936) has considered the propagation of long-period waves in connexion with the Krakatoa air wave; Pekeris (1937) and Wilkes & Weekes (1947) discussed the semidiurnal pressure oscillations in terms of long waves; and later Pekeris (1948) discussed the propagation of a pulse taking all frequencies into account, but the analysis and numerical work were not pursued to predict the exact form of the observed pulse. The present method of approach and derivation of the equations, though it arose out of a previous investigation (Scorer 1949), is fundamentally the same as that of Pekeris in the use of Fourier integrals, but differs in detail and technique. Finally, a new function had to be tabulated (Scorer 1950) before the integral giving the pulse could be evaluated.

2. Derivation of the differential equations

The values of quantities in the undisturbed atmosphere are denoted by suffix 0, the values at the ground by suffix 1, and at the tropopause by suffix 2. Plain letters are used for the disturbance; thus p_{01} is the undisturbed pressure at the ground, ρ is the disturbance of density, etc., $\rho_0+\rho$ = density, p_0+p = pressure, T_0+T = absolute temperature, $\tau_0+\tau = \dfrac{1}{T_0+T}\left(\dfrac{p_0+p}{p_{01}}\right)^{(\gamma-1)/\gamma}$ = reciprocal of potential temperature, $\varpi_0+\varpi = \dfrac{\gamma}{\gamma-1}R\left(\dfrac{p_0+p}{p_{01}}\right)^{(\gamma-1)/\gamma}$ = 'modified pressure', where R is the gas constant, and γ is the ratio of the specific heats of air. θ, ϕ, z are the spherical polar co-ordinates (see figure 1) of the point where the velocity is u, v, w. The atmosphere is assumed to be shallow so that a, the earth's radius, is written for $z+a$, z being the vertical co-ordinate measured from a level not far from the ground. In this co-ordinate system

$$\mathrm{grad} = \frac{\partial}{a\,\partial\theta},\quad \frac{\partial}{a\sin\theta\,\partial\phi},\quad \frac{\partial}{\partial z},$$

$$\mathrm{div}\,(u,v,w) = \frac{\partial(u\sin\theta)}{a\sin\theta\,\partial\theta}+\frac{\partial v}{a\sin\theta\,\partial\phi}+\frac{\partial w}{\partial z}.$$

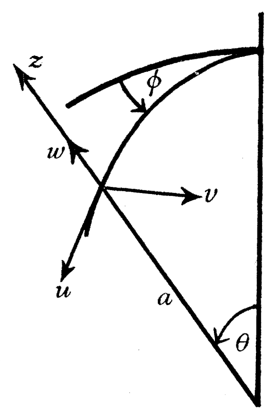

Figure 1. System of co-ordinates.

First, a periodic motion, with time factor $e^{i\sigma t}$, is considered; so that $\partial/\partial t = i\sigma$. Having established equations to determine the spatial variation of the 'modified pressure' for a given frequency σ, Fourier's integral theorem may be used to prescribe the time variation of the spatial boundary conditions which in turn are represented by a Fourier integral in k in a manner fully discussed later. k^2 is the constant of separation of the horizontal and vertical variations and is introduced to split up equation (10) below. The motion is assumed small so that products of the small quantities ρ, p, T, τ, ϖ, u, v, w and their derivatives are neglected. The equation of state is assumed, viz.

$$p_0+p = (\rho_0+\rho)\,R(T_0+T).$$

If for the moment we write differentials for the disturbance we have from the definition of ϖ given above

$$\varpi = d\varpi_0 = \frac{R}{p_{01}}\left(\frac{p_0}{p_{01}}\right)^{-1/\gamma}dp_0.$$

But
$$\tau_0 = \frac{1}{T_0} \cdot \frac{p_0}{p_{01}} \left(\frac{p_0}{p_{01}}\right)^{-1/\gamma},$$

so that
$$p = dp_0 = \frac{p_0}{\tau_0 T_0 R} d\varpi_0 = \frac{\rho_0}{\tau_0} \varpi. \tag{1}$$

Since the potential temperature is invariant in adiabatic motion

$$\frac{D}{Dt}(\tau_0 + \tau) = 0,$$

where
$$\frac{D}{Dt} = (u, v, w)\,\mathrm{grad} + \frac{\partial}{\partial t},$$

and for a small disturbance, assuming that the horizontal variation of τ_0 can be ignored,* this gives

$$i\sigma\tau + \tau_0' w = 0, \tag{2}$$

where a prime is used to denote $\partial/\partial z$ when applied to τ and ϖ, and later to χ.

The effect of the earth's rotation can be shown to be very small, the burden of the proof being that a pulse takes only a fraction of a day to pass any point on the ground. If it is neglected the equation of motion is

$$\frac{D}{Dt}(u, v, w) = -\frac{1}{\rho_0 + \rho}\,\mathrm{grad}\,(p_0 + p) + (0, 0, -g).$$

Taking the scalar product of this with (u, v, w), for small motion we obtain, using (1),

$$\frac{1}{\rho_0 + \rho}\frac{D}{Dt}(p_0 + p) = -gw + \frac{1}{\rho_0 + \rho}\frac{\partial}{\partial t}(p_0 + p)$$
$$= -gw + \frac{i\sigma}{\tau_0}\varpi. \tag{3}$$

The adiabatic equation may alternatively be written

$$\frac{D}{Dt}(p_0 + p) = c^2 \frac{D}{Dt}(\rho_0 + \rho), \tag{4}$$

where $c^2 = \gamma R T_0$; while the equation of continuity is

$$\frac{1}{\rho_0 + \rho}\frac{D}{Dt}(\rho_0 + \rho) + \mathrm{div}\,(u, v, w) = 0,$$

from which $\rho_0 + \rho$ can be eliminated by (4) and then $p_0 + p$ by (3) to give

$$\frac{1}{c^2}\left(-gw + \frac{i\sigma}{\tau_0}\varpi\right) + \mathrm{div}\,(u, v, w) = 0$$

or
$$\left(\frac{\cot\theta}{a} + \frac{\partial}{a\,\partial\theta}\right) u + \frac{\partial}{a \sin\theta\,\partial\phi} v + \left(\frac{\partial}{\partial z} - \frac{g}{c^2}\right) w + \frac{i\sigma}{c^2 \tau_0}\varpi = 0. \tag{5}$$

* This is justified on the grounds that we are not studying the motion due to an existing non-horizontal stratification of potential temperature but another small motion which is superposable and can be studied independently.

The quantities ϖ and τ have been so defined that

$$\frac{1}{\rho_0+\rho}\operatorname{grad}(p_0+p) = \frac{1}{\tau_0+\tau}\operatorname{grad}(\varpi_0+\varpi),$$

and writing this into the equation of motion, for small motion,

$$\frac{\partial}{\partial t}(u, v, w) = -\frac{1}{\tau_0+\tau}\operatorname{grad}(\varpi_0+\varpi)+(0, 0, -g),$$

so that, ignoring horizontal variations of ϖ_0, the first and second component equations are

$$i\sigma u+\frac{1}{\tau_0 a}\frac{\partial\varpi}{\partial\theta} = 0, \tag{6}$$

$$i\sigma v+\frac{1}{\tau_0 a\sin\theta}\frac{\partial\varpi}{\partial\phi} = 0, \tag{7}$$

while the third component is

$$i\sigma w+\frac{1}{\tau_0+\tau}\frac{\partial}{\partial z}(\varpi_0+\varpi)+g = 0,$$

or

$$i\sigma\tau_0 w+\frac{\partial\varpi_0}{\partial z}+\frac{\partial\varpi}{\partial z}+g(\tau_0+\tau) = 0.$$

Subtracting the hydrostatic equation (see footnote to preceding page)

$$\frac{\partial\varpi_0}{\partial z}+g\tau_0 = 0,$$

and substituting for τ in terms of w by means of (2) this becomes

$$\left(i\sigma-\frac{g\tau_0'}{i\sigma\tau_0}\right)w+\frac{1}{\tau_0}\frac{\partial\varpi}{\partial z} = 0. \tag{8}$$

Equations (6), (7) and (8) are now used to eliminate u, v, w from equation (5), and the consequent equation for the 'modified pressure' is

$$\left(\frac{\cot\theta}{a}+\frac{\partial}{a\,\partial\theta}\right)\left(\frac{i}{\sigma a\tau_0}\frac{\partial\varpi}{\partial\theta}\right)+\frac{\partial}{a\sin\theta\,\partial\phi}\left(\frac{i}{\sigma a\tau_0\sin\theta}\frac{\partial\varpi}{\partial\phi}\right)$$
$$+\left(\frac{\partial}{\partial z}-\frac{g}{c^2}\right)\left(\frac{i\sigma}{\sigma^2+g\tau_0'/\tau_0}\frac{1}{\tau_0}\frac{\partial\varpi}{\partial z}\right)+\frac{i\sigma}{c^2\tau_0}\varpi = 0. \tag{9}$$

It is now assumed that the atmosphere is sufficiently nearly horizontally stratified for the horizontal variation of τ_0 to be ignored compared with that of ϖ, i.e. that it has negligible horizontal variation in the region occupied by the pulse, then on multiplying by $\sigma\tau_0/i$ equation (9) becomes

$$\left(\frac{\cot\theta}{a}+\frac{\partial}{a^2\partial\theta}\right)\frac{\partial\varpi}{\partial\theta}+\frac{1}{a^2\sin^2\theta}\frac{\partial^2\varpi}{\partial\phi^2} = -\tau_0\left(\frac{\partial}{\partial z}-\frac{g}{c^2}\right)\left(\frac{\sigma^2}{\sigma^2\tau_0+g\tau_0'}\frac{\partial\varpi}{\partial z}\right)-\frac{\sigma^2}{c^2}\varpi. \tag{10}$$

The horizontal and vertical variations of ϖ are thus separated, and in the particular case of symmetry about the co-ordinate pole (which will be taken at the explosion) $\partial/\partial\phi = 0$, $v = 0$, and it is permissible to write

$$\varpi \equiv \varpi(z)\,\overline{\varpi}(\theta).$$

Dividing (10) by ϖ and then putting each side equal to k^2 (since the left-hand side is independent of z and the right-hand side is independent of θ) we obtain

$$\left(\cot\frac{r}{a}+\frac{\partial}{\partial r}\right)\frac{\partial\overline{\varpi}}{\partial r} = k^2\overline{\varpi}, \tag{11}$$

$$\varpi'' - \left(\frac{g\tau_0''+\sigma^2\tau_0'}{g\tau_0'+\sigma^2\tau_0}+\frac{g}{c^2}\right)\varpi' + (g\tau_0'/\tau_0+\sigma^2)\left(\frac{1}{c^2}-\frac{k^2}{\sigma^2}\right)\varpi = 0, \tag{12}$$

where r, $= a\theta$, is the great circle distance from the pole.

This treatment differs from earlier ones (Taylor 1936; Pekeris 1937) in taking ϖ as the dependent variable in the place of the divergence of velocity. This is done because ϖ is more closely related to the disturbance of pressure (equation (1)).

If suitable boundary conditions are imposed upon ϖ, equations (11) and (12) suffice to determine ϖ throughout the whole atmosphere, and the velocity is given by (6) and (8) which reduce to

$$u = \frac{i}{\sigma\tau_0}\frac{\partial\varpi}{\partial r}, \tag{13}$$

$$w = \frac{i\sigma}{\sigma^2\tau_0+g\tau_0'}\varpi'. \tag{14}$$

Equation (11) is independent of the way in which τ_0 varies with height. We next derive the form taken by equation (12) in the two layers of the model atmosphere we have chosen.

(*a*) *The troposphere.* When there is a constant lapse-rate of temperature and if the origin of z is suitably chosen

$$T_0 = -\mu\frac{(\gamma-1)g}{\gamma R}z,$$

and

$$\tau_0 = \frac{1}{T_0}\left(\frac{p_0}{p_{01}}\right)^{(\gamma-1)/\gamma},$$

$$\tau_0' = \frac{(1-\mu)\tau_0}{\mu z},$$

$$\tau_0'' = \frac{(1-\mu)(1-2\mu)\tau_0}{\mu^2 z^2},$$

where μ is a numerical constant. When $\mu = 1$ the lapse-rate is the dry adiabatic. In the troposphere $0<\mu<1$, and on substituting for c^2 and τ_0 and its derivatives and writing

$$\varpi = Z\chi, \tag{15}$$

equation (12) becomes

$$\chi'' = S\chi, \tag{16}$$

provided that

$$Z = z^{-\frac{1}{2}m}(z+b)^{\frac{1}{2}}, \tag{17}$$

where

$$\left.\begin{aligned} p &= -n+\tfrac{1}{4}(2-m),\\ n &= \frac{1-\mu}{(\gamma-1)\mu^2},\\ m &= \frac{(2\mu-1)+1/(\gamma-1)}{\mu},\\ l &= -\frac{\sigma^2}{(\gamma-1)\mu g}-\frac{(1-\mu)gk^2}{\mu\sigma^2} = -l_1-bk^2,\\ b &= \frac{(1-\mu)g}{\mu\sigma^2}, \end{aligned}\right\} \tag{18}$$

and
$$S = \frac{p}{z^2} - \frac{m/2b - l_1}{z} + \frac{\frac{3}{4}}{(z+b)^2} + \frac{m/2b}{z+b} + \left(\frac{b}{z} + 1\right) k^2. \tag{19}$$

The quantities p, n, m, l and b are determined by the lapse-rate which determines μ, by the constants γ and g, and by σ and k which determine the frequency and horizontal wave-length of the disturbance. Equation (12) has been reduced to the form (16) for purposes of numerical integration required later, so that an integration formula involving even differences of χ and χ'' only can be used.

(*b*) *The stratosphere.* The stratosphere is assumed to be isothermal at temperature T_s, with $c = c_s$, $\mu = 0$:

$$\tau_0 \propto \exp\left(\frac{(1-\gamma)g}{c_s^2} z\right),$$

$$\tau_0' = -\frac{(\gamma-1)g}{c_s^2} \tau_0,$$

$$\tau_0'' = \frac{(\gamma-1)^2 g^2}{c_s^4} \tau_0,$$

and equation (12) reduces to

$$\varpi'' - \frac{(2-\gamma)g}{c_s^2} \varpi' + \left(\sigma^2 - \frac{(\gamma-1)g^2}{c_s^2}\right)\left(\frac{1}{c_s^2} - \frac{k^2}{\sigma^2}\right)\varpi = 0, \tag{20}$$

and since the coefficients are constant, the solution is

$$\varpi = \alpha\, e^{\kappa z} + \beta\, e^{\lambda z},$$

$$\kappa, \lambda = \frac{(2-\gamma)g}{2c_s^2} \pm \left\{\left(\frac{(2-\gamma)g}{2c_s^2}\right)^2 + \left(\frac{(\gamma-1)g}{c_s^2} - \sigma^2\right)\left(\frac{1}{c_s^2} - \frac{k^2}{\sigma^2}\right)\right\}^{\frac{1}{2}}, \tag{21}$$

α and β being constants.

3. Boundary conditions; free waves; the cut-off frequency

Following Pekeris, we assume that all the oscillations considered, if they are to be set up by a source of finite energy, must possess a finite kinetic energy in a vertical column of air extending infinitely upwards. If the stratosphere continues indefinitely upwards isothermally

$$\tau_0 \propto \exp\left(\frac{(1-\gamma)g}{c_s^2} z\right),$$

$$\rho_0 \propto \exp\left(-\frac{\gamma g}{c_s^2} z\right),$$

and if ϖ, and therefore also $\tau_0 u$ and $\tau_0 w$, is proportional to $e^{\kappa z}$ or $e^{\lambda z}$ then the kinetic-energy density is proportional to

$$\rho u^2 \propto \exp\left[\pm 2z \left\{\left(\frac{(2-\gamma)g}{2c_s^2}\right)^2 + \left(\frac{(\gamma-1)g^2}{c_s^2} - \sigma^2\right)\left(\frac{1}{c_s^2} - \frac{k^2}{\sigma^2}\right)\right\}^{\frac{1}{2}}\right]. \tag{22}$$

The condition permits only the negative sign, and so in the solution of equation (20) $\alpha = 0$, and in the stratosphere
$$\varpi \propto e^{\lambda z}. \tag{23}$$

The exponent, λz, must be real for the energy to be finite. If it is unreal then it is readily shown that energy is transmitted upwards or downwards in the stratosphere; upwards transmission only can occur since no means for reflexion is assumed to exist above the tropopause, and if energy travels upwards then the lower sign is again chosen, but then no oscillation could be observed on the ground at a great distance from the source.

Free waves

If the oscillations take place over horizontal stationary ground then the boundary condition is

$$w_1 = 0, \quad \text{and it follows from equation (14) that } \varpi_1' = 0. \tag{24}$$

Equations (23) and (24) impose two boundary conditions upon the solution of (16) in the troposphere, for at the tropopause p and w are continuous, and so ϖ and $\varpi'/(g\tau_0'/\tau_0+\sigma^2)$ are continuous. But by equation (23), at the tropopause on the stratosphere side $\varpi' = \lambda\varpi$, therefore bearing in mind that τ_0'/τ_0 is discontinuous at the tropopause and expressing this condition in terms of χ, the boundary condition at $z = z_2$ imposed upon equation (16) is

$$\chi_2' = \left\{\frac{m}{2z_2} - \frac{1}{2(z_2+b)} + \lambda\frac{\sigma^2 c_s^2 - (1-\mu)(\gamma-1)g}{\sigma^2 c_s^2 - (\gamma-1)g}\right\}\chi_2. \tag{25}$$

The condition on χ at $z = z_1$, derived from (24), is

$$\chi_1' = \left\{\frac{m}{2z_1} - \frac{1}{2(z_1+b)}\right\}\chi_1. \tag{26}$$

If σ is given, then there exists a unique value of k, denoted by k^*, for which both boundary conditions are satisfied. Since the solution to (16) cannot be expressed in finite terms or in terms of tabulated functions the equation was integrated numerically from $z = z_2$, the initial conditions being made to satisfy (25), making two or more trial values of k for a given σ, and obtaining by interpolation the value of k which would satisfy (26). After some preliminary exploration it was found possible to obtain k^* with great accuracy from only two trial values.

There is, however, one disadvantage of this method of direct integration in the particular case of equation (16), namely, that the coefficient S becomes infinite when $z = -b$. This happens within the range of integration for a small fraction of the range of σ under investigation, and numerical integration is impossible in and near to this fraction. It was therefore necessary to use Pekeris's equation (1948, p. 147, eqn. (9)) for the hydrodynamical divergence for one of the values of σ, and for two other values the values of k^* already found were checked. Pekeris's equation (9) when transformed to be suitable for numerical integration is

$$\psi'' = \left[\frac{A^2-A}{z^2} - \left(B_1 + B_2\frac{k^2}{\sigma^2}\right)\frac{1}{z} + k^2\right]\psi,$$

where

$$A = \tfrac{1}{2}\left(1 + \frac{\gamma}{(\gamma-1)\mu}\right),$$

$$B_1 = -\sigma^2/(\gamma-1)\mu g,$$

$$B_2 = -(1-\mu)g/\mu,$$

$$\psi = z^A\chi,$$

and χ is now the divergence of velocity defined by Pekeris. The boundary conditions are

$$\psi_2' = \left(\nu + \frac{A}{z_2}\right)\psi_2,$$

$$\psi_1' = \left\{-g\left(\frac{k^2}{\sigma^2} - \frac{z_2\gamma}{z_1 c_s^2}\right) + \frac{A}{z_1}\right\}\psi_1,$$

$$\nu = \lambda + \frac{(\gamma-1)g}{c_s^2},$$

λ being already defined by equation (21).

The integration is slightly simpler numerically using this equation, but since the pressure is a function of the divergence and its vertical derivative, the results require more calculation to give the pulse. There is altogether nothing to choose between the use of Pekeris's equation and equation (12).

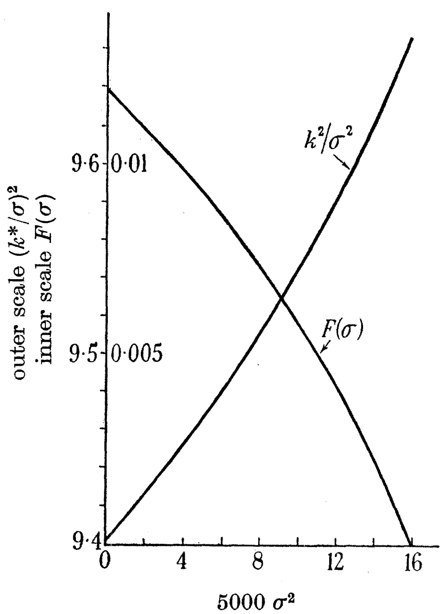

FIGURE 2. $(k^*/\sigma)^2$ and $F(\sigma)$ as functions of σ^2.

For convenience $(k^*/\sigma)^2$ was found as a function of σ^2 and the results are given in figure 2 and table 1, for the atmosphere with the following constants:

$$g = 980{\cdot}6\,\text{cm.sec.}^{-2};\quad T_2 = T_s = 229{\cdot}53^\circ\,\text{K};\quad T_1 = 286{\cdot}91^\circ\,\text{K};$$

$$z_2 - z_1 = 9{\cdot}6137\,\text{km.};\quad \gamma = 1{\cdot}403.$$

In the numerical work the unit of length was taken as 0·9806/1·02 km.; when the depth of the troposphere was 10 units, $g = 1{\cdot}02$, $c_s^2 = 10^{-1}$, and $\dfrac{1-\mu}{\mu} = 0{\cdot}64424$.

In finding k^* as a function of σ for the free waves, that is, waves propagated over level ground, no reference is made to the form of equation (11). If in the place of a spherical earth the ground had been taken as an infinite plane and the motion assumed to be independent of one horizontal co-ordinate y, then (11) takes the form

$$\frac{\partial^2 X}{\partial x^2} + k^2 X = 0, \tag{27}$$

where $$\varpi \equiv X(x) . \varpi(z),$$

and (12) is also satisfied. In this case the horizontal wave-length of the oscillations is $2\pi/k$, and σ/k^* is the horizontal velocity of the free waves. The meaning of k in equation (12) becomes clear only in terms of the solution of equation (11) and the way in which the explosive source is defined in terms of that solution.

Table 1

The unit of length for the 2nd and 3rd columns is $\frac{0{\cdot}9806}{1{\cdot}02}$ km. Three more significant figures were obtained for k^*/σ than are given in the table. The cut-off group velocity was obtained by extrapolation.

	free waves		
$5000\,\sigma^2$	phase veloc.$^{-1}$ k^*/σ	group veloc.$^{-1}$ $dk^*/d\sigma$	relative intensity $F(\sigma) \times 10^3$
0	3·0663	3·0663	11·918
1	3·0682	3·0721	11·410
2	3·0702	3·0783	10·897
3	3·0723	3·0849	10·363
4	3·0744	3·0919	9·803
5	3·0766	3·0994	9·200
6	3·0790	3·1074	8·611
7	3·0814	3·1159	7·973
8	3·0839	3·1251	7·277
10	3·0893	3·1457	5·786
12	3·0952	3·1699	4·103
14	3·1018	3·1988	2·173
15·940	3·1089	3·231	0

Cut-off frequency

The k, σ plane is divided into two regions in which the condition

$$\left(\frac{(2-\gamma)\,g}{2c_s^2}\right)^2 + \left(\frac{(\gamma-1)\,g}{c_s^2} - \sigma^2\right)\left(\frac{1}{c_s^2} - \frac{k^2}{\sigma^2}\right) > 0 \tag{28}$$

is or is not satisfied. We are concerned only with the region in which it is satisfied for, as explained above, the oscillations must possess finite kinetic energy. The curve $k = k^*(\sigma)$ is found to lie partly in each region. For long waves ($\sigma = 0$) it is known from the work of Taylor and others that there is only one value of k^*, there being for this model atmosphere only one mode of oscillation. The present numerical exploration confirms that of Pekeris, that for each σ there is only one value of k^* and that there is a 'cut-off' frequency σ_c such that only when $\sigma < \sigma_c$ is (28) satisfied when $k = k^*$.

The physical meaning of this is that if they are excited by a source on the ground, oscillations of frequency less than σ_c are reflected down at the tropopause, while the energy of those of greater frequency is transmitted indefinitely upwards in the stratosphere. It is in this property that our model atmosphere differs from the actual atmosphere which is known to have warmer layers at heights above 30 km. capable of trapping oscillations of audible frequencies. The results derived from the long-wave end of the spectrum only will bear good resemblance to observations.

It is found that the velocity σ/k^* of the free waves in our model atmosphere is equal to the velocity of sound c at some level in the troposphere depending upon σ. Their speed is slower than that of sound at the ground but faster than that in the stratosphere. The actual period at the cut-off frequency is found to be

$$2\pi/\sigma_c = 111{\cdot}28\,\text{sec.}$$

4. The dispersion of a pulse

It now remains only to impose a boundary condition at the ground which will correspond to an explosion. The pole ($r = 0, \theta = 0$) of the co-ordinates is taken as the centre of the explosion, which is represented by an upward movement of the ground over a finite (or infinitesimal) region for a finite (or infinitesimal) time so that a finite calculable volume of ground is introduced into the atmosphere. This corresponds to the sudden creation of gas by high explosive, the emission of gas or steam by a volcano, the sudden introduction into the atmosphere of a meteorite and its aura, or to the sudden generation of heat which causes a large and rapid expansion. The essential characteristic of an explosion for the present purpose is the rapid introduction of a volume into the atmosphere, and it will be seen that provided it is rapid and local enough, the pulse observed at a great distance depends very little upon the exact time taken to introduce it or the horizontal extent of the source.

We make use of the Fourier integral

$$w_1 = B\int_0^\infty d\sigma \int_0^\infty dk\, \Sigma(\sigma, t)\, K(k, r)\, \mathrm{e}^{ikr}\,(\mathrm{e}^{i\sigma t} + \mathrm{e}^{-i\sigma t}),$$

$$\text{(real part only)} \tag{29}$$

which, by a suitable choice of the functions Σ and K gives to w_1 a finite value near to $r = 0$, $t = 0$, and a negligible value elsewhere. The boundary condition (29) which defines the vertical velocity of the air at the ground thus represents a rapid, local elevation of the ground introducing a known volume into the atmosphere. B is a constant used to determine the magnitude.

Before inserting specific functions Σ and K we derive the formula for the pressure pulse p_1 observed at the ground at a great distance (large r). If

$$w = W(z), \quad \varpi = \Phi(z) \tag{30}$$

are corresponding solutions of equations (12) and (14) which satisfy the upper-boundary condition, then the vertical velocity at any height, with equation (28) as the lower-boundary condition, is

$$w = B\int_0^\infty d\sigma \int_0^\infty dk\, \Sigma K\, \mathrm{e}^{ikr}(\mathrm{e}^{i\sigma t} + \mathrm{e}^{-i\sigma t})\,\frac{W}{W_1},$$

so that by equation (30)

$$\varpi = B\int_0^\infty d\sigma\int_0^\infty dk\,\Sigma K\,\mathrm{e}^{ikr}(\mathrm{e}^{i\sigma t}+\mathrm{e}^{-i\sigma t})\frac{\Phi}{W_1}. \tag{31}$$

For free waves $k = k^*(\sigma)$, $W_1 = 0, \Phi \neq 0$, so that the integration with respect to k may be performed by the method of residues, k^* being the pole of the integrand. The disturbance of pressure at the ground ($z = z_1$) is obtained from (31) and (1), giving

$$p_1 = B\frac{\rho_{01}}{\tau_{01}}\int_0^\infty d\sigma\,\Sigma(\mathrm{e}^{i\sigma t}+\mathrm{e}^{-i\sigma t})\left\{\int_\Gamma dkK\frac{\Phi_1}{W_1}\mathrm{e}^{ikr}+\pi i\,\mathrm{e}^{ik^*r}K^*\Big/\left[\frac{\partial}{\partial k}\frac{W_1}{\Phi_1}\right]_{k=k^*}\right\}, \tag{32}$$

Γ being a contour in the upper half k-plane from the origin to $+\infty$ but not coinciding with the real axis except at its end-points. The first term in the bracket, though possibly appreciable near the source ($r = 0$) becomes negligible for large r on account of the oscillatory nature of e^{ikr} and negative real part of ikr, while $K(\Phi_1/W_1)$ is a slowly varying function of k for all r. For values of σ which do not give a real value of k^*, i.e. for $\sigma > \sigma_c$, the value of k^* has not been found, but for large r the factor e^{ik^*r} renders the contribution to p_1 negligible. The first term, together with the second term for $\sigma > \sigma_c$, represents a pulse which is perceptible at the ground near the source only, and includes the component oscillations of all those frequencies which are not trapped at the tropopause. The pulse observed on the ground at a great distance is therefore

$$p_1 = \pi iB\frac{\rho_{01}}{\tau_{01}}\int_0^{\sigma_c} d\sigma(\mathrm{e}^{i\sigma t}+\mathrm{e}^{-i\sigma t})\,\mathrm{e}^{ik^*r}\,\Sigma K\Big/\left[\frac{\partial}{\partial k}\frac{W_1}{\Phi_1}\right]_{k=k^*}. \tag{33}$$

Now by equations (11) and (30)

$$\begin{aligned}\left[\frac{\partial}{\partial k}\frac{W_1}{\Phi_1}\right]_{k=k^*} &= \left[\frac{\partial}{\partial k}\frac{i\sigma\Phi_1'}{(g\tau_{01}'+\sigma^2\tau_{01})\Phi_1}\right]_{k=k^*}\\ &= \frac{i\sigma}{g\tau_{01}'+\sigma^2\tau_{01}}\left[\frac{\partial}{\partial k}\left(\frac{\chi_1'}{\chi_1}\right)\right]_{k=k^*} \quad \text{by (15)}\\ &= \frac{4i}{\tau_{01}}\Big/F(\sigma),\end{aligned} \tag{34}$$

where
$$2/F(\sigma) = \frac{k/\sigma}{g\tau_{01}'/\tau_{01}+\sigma^2}\left[\frac{\partial}{\partial(k^2/\sigma^2)}\left(\frac{\chi_1'}{\chi_1}\right)\right]_{k=k^*}. \tag{35}$$

$F(\sigma)$ is the relative intensity in the pulse of the component oscillations of frequency σ, in the case $\Sigma = K = 1$. The quantity in the square bracket is evaluated numerically in the course of finding k^* by making trial values of k^2/σ^2 for a given σ^2, and $F(\sigma)$ is given in figure 2 and table 1 as a function of σ^2. The numerical integration of equation (16) was performed from the tropopause downwards because it is required in (35) that χ_1 shall arise from a function Φ, given by (30), satisfying the upper-boundary condition. We now have

$$p_1 = \tfrac{1}{4}\pi\rho_{01}B\int_0^{\sigma_c} F\Sigma K^*\,\mathrm{e}^{ik^*r}\,(\mathrm{e}^{i\sigma t}+\mathrm{e}^{-i\sigma t})\,d\sigma, \tag{36}$$

which gives p_1 as a function of r and t.

The succeeding section discusses some particular forms of Σ and K.

5. The form of the explosive source

The region over which $w_1 \neq 0$ is only a small fraction of the earth's surface, and can be considered flat; so that analytically the source has the same form as if it were on an infinite plane earth. In such circumstances if the oscillations were independent of the horizontal co-ordinate y, an instantaneous line source such that w_1 is zero at all points except $x = 0$ and at all times except $t = 0$, but is integrably infinite at $x = 0, t = 0$, is represented by equation (29) with

$$\Sigma = K = 1, \quad r = x, \tag{37}$$

giving
$$w_1 = 2B\int_0^\infty \cos\sigma t\, d\sigma \int_0^\infty \cos kx\, dk, \tag{38}$$

$$p_1 = \tfrac{1}{4}\pi\rho_{01} B\int_0^{\sigma_c} F(\sigma)\, e^{ik^*x}(e^{i\sigma t} + e^{-i\sigma t})\, d\sigma \quad \text{(real part)}. \tag{39}$$

It may be noted that $\cos kx$ is a solution of (26), $2\pi/k$ is the horizontal wave-length and σ/k the horizontal phase velocity of the component oscillations.

When r is small equation (11) becomes Bessel's equation of zero order, and an instantaneous point source at $r = 0$, $t = 0$ is represented by the well-known integral (used by Pekeris)

$$w_1 = 2B\int_0^\infty \cos\sigma t\, d\sigma \int_0^\infty J_0(kr)\, k\, dk. \tag{40}$$

The volume introduced is

$$\int_0^\infty dt \int_0^\infty 2\pi r w_1\, dr = 8\pi^2 B$$

(Lamb 1932, art. 102) and

$$p_1 = \tfrac{1}{4}\pi\rho_{01} B\int_0^{\sigma_c} F(\sigma)\, J_0(k^*r)\, k^*(e^{i\sigma t} + e^{-i\sigma t})\, d\sigma. \tag{41}$$

Here $K\,e^{ikr}$ in (29) has been replaced by $J_0(kr)\,k$, and the argument used to dispose of the first term in (33) is valid because the Bessel function oscillates like the cosine. When r is sufficiently large, either the integrand is negligible because k^* is small when σ is near to 0 or else $J_0(k^*r)$ may be replaced by its asymptotic form, giving

$$p_1 = \tfrac{1}{4}\pi\rho_{01} B\int_0^{\sigma_c} F(\sigma)\left(\frac{2k^*}{\pi r}\right)^{\frac{1}{2}} e^{i(k^*r - \frac{1}{4}\pi)}\,(e^{i\sigma t} + e^{-i\sigma t})\, d\sigma. \tag{42}$$

However, when r is large, Bessel functions are no longer solutions of equation (11) and it is therefore necessary to replace $r^{-\frac{1}{2}}$ by $(a\sin\theta)^{-\frac{1}{2}}$, thus substituting the asymptotic form of the solution of equation (11) (Jeffreys & Jeffreys 1946, art. 24·15) for the asymptotic form of the Bessel function. The factor $k^{*\frac{1}{2}}$ means that as compared with a line source, a point source favours the shorter wave-lengths. The form taken by (42) on a spherical earth is obtained by putting into the expression (36) the values

$$\Sigma = 1, \quad K = \left(\frac{2k}{a\sin\theta}\right)^{\frac{1}{2}} e^{-\frac{1}{4}i\pi}. \tag{43}$$

This is the form of source for which the pulse form at various distances has been calculated. The modification required when the source is diffused in space or time is illustrated by three examples, in each of which the relative intensity of that part of the pulse due to the longer waves is increased. It is reasonable to suppose that this would be the effect of most kinds of spreading of the source in space or time. Since the longer waves travel faster the effect is simply to intensify the earlier part of the pulse at the expense of the later part.

(*a*) A source of intensity gradually increasing to a maximum at $t = 0$ and then dying away, the strength at time t being proportional to $U/(U^2+t^2)$, which is the real part of $\int_0^\infty e^{-\sigma U + i\sigma t}\, d\sigma$, is obtained by putting $\Sigma = e^{-\sigma U}$, where U is a constant, being the time taken for the source to decrease from its maximum to half its maximum strength.

(*b*) If the intensity is proportional to $\cos st$ in $-\pi/2s < t < \pi/2s$ and is zero at all other times, we write

$$\Sigma = \frac{2s}{s^2-\sigma^2}\cos\frac{\pi\sigma}{2s}. \tag{44}$$

This again is a decreasing function of σ when σ is positive.

(*c*) If instead of a point source the intensity is uniform inside $r = b$ and zero outside we find (cf. Lamb 1932, art. 102) that it is necessary to write

$$K = \frac{2}{\pi b} J_1(kb)\left(\frac{2}{k\pi a \sin\theta}\right)^{\frac{1}{2}} e^{-\frac{1}{4}i\pi} \tag{45}$$

which multiplies the factor given in (43) by $2J_1(kb)/kb$. The relative intensity of the long waves is again increased.

In these instances the constants U, s and b must be chosen so that Σ and K^* are slowly varying functions of σ, as is $F(\sigma)$, for upon that fact depends the method of evaluating the pulse numerically.

6. Evaluation of the integral $I = \int_0^{\sigma_c} F\Sigma K e^{ikx}(e^{i\sigma t} + e^{-i\sigma t})\, d\sigma$ (equation (36))

It is necessary to describe only briefly how the method given elsewhere (Scorer 1950) was used to evaluate the integral (36).

Henceforth we write k for k^*, K for K^*, and k' for $dk^*/d\sigma$.

It became evident as the values of k^2/σ^2 were found, and plotted against σ^2, that as $\sigma \to 0$ then $k'' \to 0$ also, and application of the principle of stationary phase, suggested by Pekeris for this problem, could not be made in the usual way,† for it is only justifiable if

$$|k'''| << |k''|^{\frac{3}{2}}. \tag{46}$$

But since $k^{\mathrm{iv}} \to 0$ as $\sigma \to 0$, the method is adequate if $kx - \sigma t$ is expressed as a cubic in σ instead of the usual quadratic.

Thus if $k(\sigma_0) = k_0$ is the solution of

$$k'x - t = 0 \tag{47}$$

† As given, for instance, in Lamb (1932, art. 241).

($k'x+t=0$ has in the present case no solution, so that the term in $e^{i\sigma t}$ has no point of stationary phase and is therefore neglected), and

$$k = k_0+(\sigma-\sigma_0)\,k_0'+\tfrac{1}{2}(\sigma-\sigma_0)^2 k_0''+\tfrac{1}{6}(\sigma-\sigma_0)^3 k''', \tag{48}$$

then

$$kx-\sigma t = a+b\sigma+c\sigma^2+d\sigma^3, \tag{49}$$

where

$$\left.\begin{aligned} a &= (k_0-\sigma_0 k_0'+\tfrac{1}{2}\sigma_0^2 k_0''-\tfrac{1}{6}\sigma_0^3 k_0''')\,x,\\ b &= (-\sigma_0 k_0''+\tfrac{1}{2}\sigma_0^2 k_0''')\,x,\\ c &= (\tfrac{1}{2}k_0''-\tfrac{1}{2}\sigma_0 k_0''')\,x,\\ d &= \tfrac{1}{6}k_0'''\,x. \end{aligned}\right\} \tag{50}$$

Thus, given x, by (47), σ_0 is a function of t, and following the method given by Scorer (1950) we obtain

$$I = \frac{\pi}{l}F_0\Sigma_0 K_0 \exp\left\{i\left(a-\frac{bc}{3d}\right)\right\} L(z), \tag{51}$$

which gives I as a function of σ_0 and therefore of t for a given x, where

$$\left.\begin{aligned} l &= (3d)^{\frac{1}{3}},\\ z &= (3d)^{-\frac{1}{3}}\,(b-c^2/3d), \end{aligned}\right\} \tag{52}$$

and

$$\begin{aligned} L(z) &= \operatorname{Ai}(z)+i\operatorname{Gi}(z)\\ &= \frac{1}{\pi}\int_0^\infty \exp\{i(uz+\tfrac{1}{3}u^3)\}\,du. \end{aligned} \tag{53}$$

Tables of $\operatorname{Ai}(z)$ are given by Miller (1946) and of $\operatorname{Gi}(z)$ by Scorer (1950).

To evaluate the derivatives of k most accurately by numerical methods we write

$$k^2/\sigma^2 = V(\sigma^2), \quad V^{(n)} = \frac{d^nV}{d(\sigma^2)^n}, \tag{54}$$

and obtain

$$\left.\begin{aligned} k/\sigma &= V^{\frac{1}{2}},\\ k &= \sigma V^{\frac{1}{2}},\\ k' &= V^{\frac{1}{2}}+\sigma^2 V^{-\frac{1}{2}}V',\\ k'' &= \sigma V^{-\frac{1}{2}}\{3V'-\sigma^2V^{-1}V'^2+2\sigma^2V''\},\\ k''' &= V^{-\frac{1}{2}}\{3V'+12\sigma^2V''+4\sigma^4V'''-6\sigma^2V^{-1}(V'^2+\sigma^2V'V'')+3\sigma^4V^{-2}V'^3\}. \end{aligned}\right\} \tag{55}$$

V', V'', V''' were obtained by numerical differentiation of $V(\sigma^2)$. This procedure, though most undesirable from the computation point of view, was unavoidable, but the higher derivatives of V only made small contributions in the formulae (55). This was the main reason for calculating k^2/σ^2 at equal intervals of σ^2, for thereby a very smooth function is obtained.

Equations (47) and (51) were then used to obtain I and therefore p_1, as a function of t for certain chosen values of r.

7. Comparison of results with observation

The pulses to be observed at five different great circle distances from the explosion are given in figures 3 to 7. The units of pressure are microbars (dyne cm.$^{-2}$) per cu.km. of explosion. If all the energy to cause the expansion were supplied as heat the amount per cu.km. would be approximately $3{\cdot}8 \times 10^{21}$ ergs, independent of the volume to which the heat is applied, assuming that the heating took place at constant pressure. If the volume is introduced mechanically the energy required depends very much upon the rate of introduction, but it is probably of the same order of magnitude as if the energy is supplied as heat if the expansion is rapid enough to be termed an explosion. Details of these calculated pulses are given in table 2, the time zero in each case being placed at the time of arrival of long waves. The time of cut-off is the time of

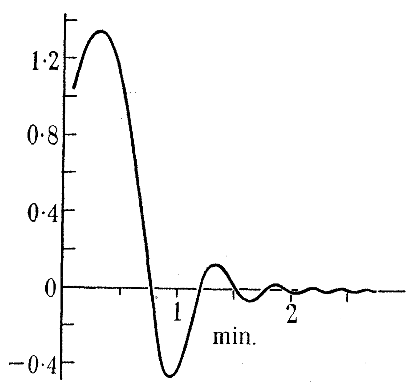

Figure 3

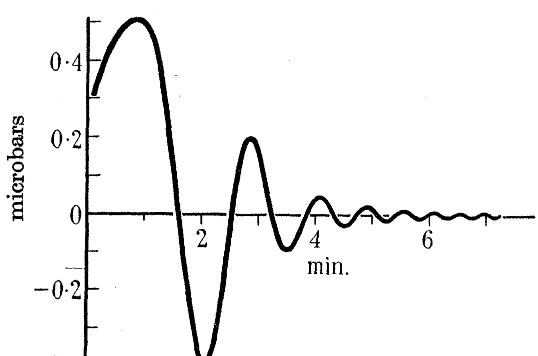

Figure 4

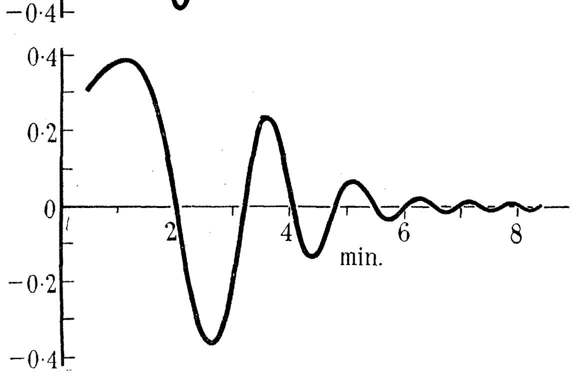

Figure 5

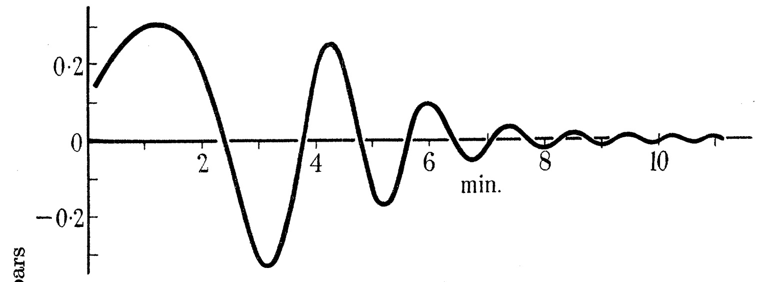

FIGURE 6

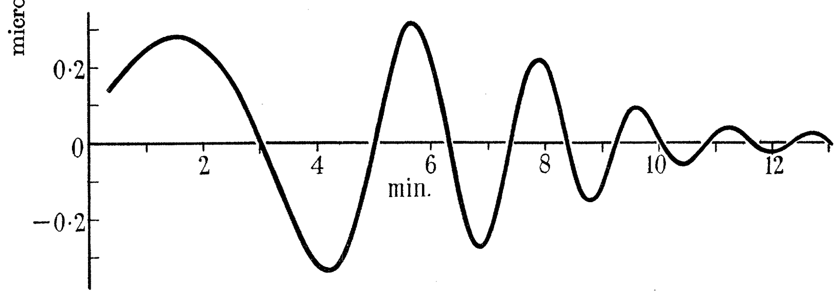

FIGURE 7

FIGURES 3 to 7. The pulses at five different great circle distances from the explosion.

arrival of waves travelling with the group velocity corresponding to the cut-off frequency. The amplitude of the first five half-oscillations is the difference in pressure between successive stationary values. Only in figure 3 is the complete pulse form given; in the other cases the oscillations continue with decreasing amplitude and increasing frequency to a period of 12·7 sec. at the cut-off time. The form of the pulse before the first crest is uncertain for the stationary phase method is inadequate near $\sigma_0 = 0$ because the factor $k^{\frac{1}{2}}$ in the integrand is rapidly varying near $k = 0$; but in any event, an instrument recording the pulse would be fitted with some form of leakage so that slow changes in pressure are not shown. Thus the first crest is probably too low on the microbarograph traces for the English stations shown with fourfold magnification in figures 8 to 11. The rise in pressure from the first trough to the second crest has been taken as well as the fall from the first crest to the first trough when theory and observation are compared to give an estimate of the volume of the explosion of the Siberian meteorite, except in the case of stations around 970 km. distant when the fall from the first crest to the first trough only is taken. In table 3 are briefly set out the characteristics of the best available observations. The Siberian stations were twelve in number ranging from 660 to 1230 km. distant. The average distance was 990 km. The pressure-pulse amplitude varied from 0·40 to 2·45 and was not well related to the distance. This information is given in the paper by Astapowitsch (1934), whence also figure 12 was taken, showing the pulse observed near Leningrad.

There appears to be a loss of well over half the energy of the pulse between the Siberian and English stations. This may be attributed to various causes besides mechanical dissipation; the prevailing wind and temperature regime may cause refraction and distortion of the wave fronts; and the siting of the station relative to these and to mountain ranges may influence the pulse received. An expansion of 1000 cu.km. would be caused by $3{\cdot}8 \times 10^{24}$ ergs if all the energy were supplied as heat, whereas no previous estimate of the energy exceeds 5×10^{21} ergs.

In this estimate we have calculated the energy required to be supplied as heat in order to produce the observed pulse, but only a fraction of this energy passes into the kinetic energy of the pulse. The energy of all those component frequencies for which $\sigma > \sigma_c$ escapes into the upper stratosphere and is dissipated by viscosity. Work must be done merely to introduce the volume: approximately 10^{24} ergs would be required to introduce 1000 cu.km. so slowly that no pulse were created at all. Previous estimates, summarized by Astapowitsch, of the energy depend on calculating the energy of the air wave or the work done in felling the observed number of trees, that is, the

FIGURE 8. Meteorological Office.

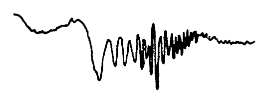

FIGURE 9. Reading.

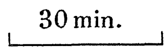

FIGURE 10. Petersfield.

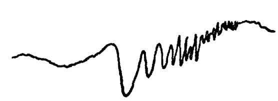

FIGURE 11. South Kensington.

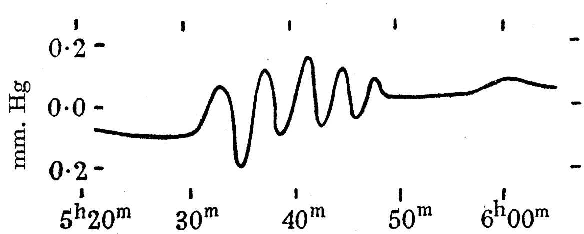

FIGURE 12. Diagram of record of air waves caused by the explosion on the occasion of the fall of the meteorite on 30 June 1908 according to the records of the Sprung microbarograph at the Slutsk Geophysical Observatory near Leningrad.

energy required for one of the many phenomena produced, but no estimate was made of what proportion of the total energy was expended upon it. Alternatively, they depend upon comparison with the Krakatoa eruption for which no more reliable estimate exists, or with earthquakes. We are concerned here with the total amount of energy communicated to the atmosphere; and since the energy of the earthquake shock must be added to this it appears that the great magnitude of the phenomenon has not hitherto been appreciated.

The time interval between successive nodes is not in good agreement with observation. The atmosphere on the occasion seems to have been more dispersive than the model chosen, but even so no explanation is offered as to why the oscillations at Leningrad should have been slower than those observed in England.

Figure 13 shows, in an enlarged scale, the form of the pulse observed in England. It was derived by Whipple (1930) from six microbarograms, four of which are given

TABLE 2. CHARACTERISTICS OF THE CALCULATED PULSES

figure	great circle distance in km.	interval between explosion and time zero min. sec.	time of cut-off min. sec.	amplitude of successive half-oscillations (microbars)	time interval between successive nodes (sec.)	no. of complete oscillations before cut-off
3	961	51 6	2 45	1·80, 0·59, 0·19, 0·08, 0·03	66, 26, 19, 15, 12, 10	5
4	3650	194 1	10 29	0·88, 0·57, 0·29, 0·14, 0·08	98, 56, 41, 36, 29, 25	19
5	5607	298 2	16 7	0·74, 0·59, 0·36, 0·19, 0·10	122, 71, 51, 46, 39, 33	30
6	7691	408 50	22 6	0·64, 0·58, 0·42, 0·26, 0·15	144, 83, 61, 49, 47, 41	42
7	13290	706 28	38 11	0·62, 0·66, 0·60, 0·50, 0·37	177, 124, 80, 65, 56, 51	70

TABLE 3. CHARACTERISTICS OF THE PULSES OBSERVED ON THE OCCASION OF THE FALL OF THE GREAT SIBERIAN METEORITE IN 1908

station	distance (km.)	time to first trough (min.)	amplitude of successive half-oscillations	Time interval between successive nodes (min.)	volume of explosion derived from 1st half oscillation (cu.km.)	volume of explosion derived from 2nd half oscillation (cu.km.)
Siberian	(average) 990	—	1·15 mm. mercury	—	880	—
Leningrad (Slutsk)	3740	195·2	0·275, 0·275, 0·25, 0·21, 0·21 mm. mercury	2·0, 2·0, 2·0, 1·9, 1·8, 1·6	430	660
English	(average) 5752	(average) 301	197, 214, 101, 73, 62 microbars	2·5, 1·3, 1·0, 1·0, 1·0, 1·0	270	360

in figures 8 to 11. The sudden and more rapid oscillations which are completely absent from the calculated curves may reasonably be supposed to be due to the warm layers above 30 km., which have not been taken into account. Whipple remarks that these waves travelled considerably faster than the sound waves known on occasions to have been reflected by these layers, but then only a single reflexion took place, and the height of reflexion was a considerable fraction of the distance between the explosion and the observer; the propagation was thought of in terms of rays of sound which were inclined to the ground. The oscillations which are observed at a great distance may not legitimately be studied by geometrical optics which is suitable for high-frequency oscillations only; gravity now becomes all important, and the wave-fronts are vertical; the path of a wave-front element from the region of the explosion to the observer is horizontal, and it is not surprising to find a greater horizontal component to the velocity of propagation than in the case of a single reflexion at high levels. The labour involved in the numerical work necessary to study this has so far been prohibitive. All that can be concluded on the basis of the present work is that this sudden, second, set of oscillations was probably not due to a separate explosion because at such a great distance the pulses for two separate explosions would be identical in form.

The kinetic-energy density of the free waves dies away exponentially upwards in an isothermal stratosphere when $\sigma < \sigma_c$, but dies away more slowly as σ increases until at $\sigma = \sigma_c$ it is constant at all heights; $F(\sigma)$ therefore decreases to zero as σ increases to σ_c, for at $\sigma = \sigma_c$ the energy of the oscillations is infinite unless the amplitude is zero. If the warmer layers at higher levels were taken into account then the cut-off would not occur until a higher value of σ were reached; therefore $F(\sigma)$ would assume a greater value for those frequencies which make up the later part of the calculated pulse, and the amplitude of the oscillations would die away less rapidly. This may, in part, explain the difference in form between the calculated and observed pulses. No such ready argument has been found to deduce the effect of the warm upper layers of the stratosphere on the period of the oscillations in the later part of the pulse.

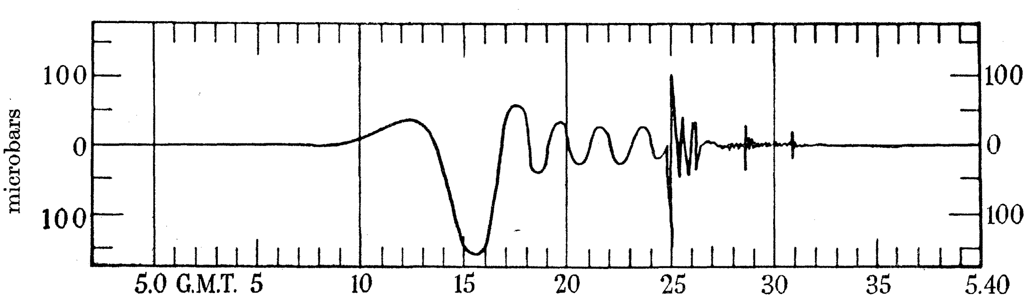

FIGURE 13.

Comparison with the Krakatoa air wave is difficult because there was probably not a single sudden explosion, nor were instruments set up capable of displaying the more rapid oscillations. There was undoubtedly one explosion greater than the others, but the others were great enough to produce observable pulses. The interval between the explosions was of the same order of magnitude as the time taken for one pulse to pass a distant point, so that the separate pulses interfered. The best observations were, furthermore, made at stations of very different latitude from each other and from the eruption, so that the pulse form would be distorted by passage through regions with large variations in wind and temperature structure.

Finally, two remarks may be made concerning the calculated pulse form. The local period of oscillation at any part of the pulse does not correspond to the period of the waves, travelling with the group velocity and arriving at that instant. The cut-off frequency has a period of 111·28 sec. which is nearly ten times the period of the oscillations at the termination of the pulse. Pekeris found a cut-off period of about 2 min. for a slightly different model atmosphere.

It has been assumed that propagation takes place without loss of energy, and the amplitude at 7691 km. (figure 6) is smaller than at 13290 km. (figure 7) because the wave front in the former instance is extended round an equatorial belt (the explosion being taken as pole). It contracts subsequently to a small zone of higher latitude and the amplitude increases.

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